Mathematics with ICT in Key Stage 3

Magic squares

Introduction

The introduction to the document *Integrating ICT into mathematics in Key Stage 3* states:

> *Computers offer powerful opportunities for pupils to explore mathematical ideas, to generalise, explain results and analyse situations, and to receive fast and reliable, and non-judgemental, feedback. Their use needs careful planning – not just of the organisation of hardware and appropriate software but also of activities that allow for off-computer mathematical thinking as well as on-computer exploration.*

Mathematics with ICT in Key Stage 3 comprises sets of teaching notes, lesson plans and resources. Lesson content has been developed through a flexible use of the *Sample medium-term plans for mathematics*. Objectives are drawn from both the *Framework for teaching mathematics: Years 7, 8 and 9* and the *Framework for teaching ICT capability: Years 7, 8 and 9.*

'Magic squares' uses spreadsheets to explore the mathematics behind magic squares. It relates particularly to the Year 7 Algebra 2 (7A2) unit in the autumn term of Year 7 in the *Sample medium-term plans for mathematics*.

In planning these lessons:

- the mathematics objectives are drawn from 'Algebra' and 'Using and applying mathematics to solve problems' in the mathematics Framework;
- the ICT objectives are from 'Models and modelling' and 'Analysing and automating processes' in the ICT Framework.

Using spreadsheets in the mathematics classroom

Integrating ICT into mathematics in Key Stage 3 comments on the use of spreadsheets:

> *Spreadsheets allow users to sort and carry out a vast range of calculations on lists and arrays of numbers. The data contained in the spreadsheet can also be represented in various graphs and charts. There are a number of uses for spreadsheets in mathematics in Key Stage 3:*
>
> - *generating and exploring number patterns and sequences;*
> - *solving simple optimisation and number problems;*
> - *developing an appreciation for the concept of a variable;*
> - *constructing and exploring functions, including equivalence of functions;*
> - *analysing data and statistics.*
>
> *It is important to note that the notation used for constructing mathematical functions is often different to mathematical conventions. Also, some mathematical graphs and diagrams are difficult to produce correctly.*

The report *ICT and mathematics: a guide to learning and teaching mathematics 11–19* contains further relevant information – see especially pages 36–39.

Outline of lessons

Using a spreadsheet to explore magic squares (7A2.1)

This lesson is designed to be used as part of the teaching programme for mathematics in the autumn term. Pupils use a spreadsheet to explore the structure of a simple magic square, with use of formulae to produce totals of rows, columns and diagonals. The construction of these formulae is modelled initially by the teacher and then continued by the pupils. Once the model is in place, pupils explore its properties before changing the numbers to create different magic squares.

Extension activities and follow-up lessons (7A2.2 and 7A2.3)

In subsequent lessons, pupils are challenged to use their created templates to invent new magic squares with different 'magic numbers'. In doing this, they discuss the generic properties of 3 by 3 magic squares. As an extension, pupils can construct spreadsheet grids for different patterns and arrangements, always working towards matching totals, and discuss their properties.

Lesson objectives

Mathematics objectives

- Use simple formulae from mathematics and other subjects; substitute positive integers into simple linear expressions and formulae and, in simple cases, derive a formula.
- Present and interpret solutions in the context of the original problem; **explain and justify methods and conclusions**, orally and in writing.
- **Use logical argument to establish the truth of a statement**; give solutions to an appropriate degree of accuracy in the context of the problem. (Year 8)
- Use letter symbols to represent unknown numbers or variables; know the meanings of the words *term*, *expression* and *equation*.
- Understand that algebraic operations follow the same conventions and order as arithmetic operations.

ICT objectives

- Use software to investigate and amend a simple model by entering rules or formulae and checking their appropriateness and accurate working.
- Develop ICT-based models and test predictions by changing variables and rules. (Year 8)

Resources

Main ICT resources

- Computer with large-screen facility for whole-class display
- Spreadsheet software
- Computer suite

Other resources

- Microsoft® Excel spreadsheet files ‘Magic square’ and ‘It adds up’
- Resources to be displayed or printed:
  7A2.1 Building a magic square in Excel
  7A2.2 Excel helpsheet: formatting cells
  7A2.3 More puzzles

Prerequisites

Pupils’ prior learning

Pupils should already know how to:

- order, add and subtract positive numbers;
- recognise odd, even and square numbers.

ICT expertise needed by the teacher

To teach these lessons you will need to know how to:

- enter values and formulas into a spreadsheet.

Preparation and planning

Print copies of any of the resources you need.

Key vocabulary

algebra
equals
explain
expression
formatting
formula
integer
magic number
prove
reason
simplify
sum
symbol
total
value
variable

7A2.1 Using a spreadsheet to explore magic squares

Starter

Allow 10 minutes

Vocabulary

equals, explain, formatting, formula, integer, sum, total, value

Resources

Excel file 'Magic square'

One computer for every two or three pupils

Optional: interactive whiteboard

ICT objective

- Use software to investigate and amend a simple model by entering rules or formulae and checking their appropriateness and accurate working.

Display the file 'Magic square' and explain the magic square puzzle.

Invite pupils to change values within the square, using the numbers 1 to 9 once only, to attempt to obtain row, column and diagonal totals that are equal.

Encourage discussion about ways of doing this by adding three numbers.

Tell pupils that they are going to work on this problem but that they will need to produce a simple magic square of their own on a spreadsheet so that they can search for a solution.

Main teaching

Allow 35 minutes

Vocabulary

magic number, reason, sum, total

Resources

7A2.1 'Building a magic square in Excel'

7A2.2 'Excel helpsheet: formatting cells'

7A2.2 'More puzzles' (optional extension work)

Mathematics objectives

- Use simple formulae from mathematics and other subjects; substitute positive integers into simple linear expressions and formulae and, in simple cases, derive a formula.
- **Use logical argument to establish the truth of a statement**; give solutions to an appropriate degree of accuracy in the context of the problem. (Year 8)

ICT objectives

- Use software to investigate and amend a simple model by entering rules or formulae and checking their appropriateness and accurate working.
- Develop ICT-based models to test predictions by changing variables and rules.

Model the first steps in creating the magic square spreadsheet file, explaining how to construct a formula in a spreadsheet and as much of the formatting as you wish pupils to understand at this stage – use **helpsheet 7A2.2** as a reminder if you wish. Pupils work in pairs at computer to produce and then use their magic squares to investigate possible solutions. You may wish to refer to aspects of **resource 7A2.1**, which provides step-by-step support through this process

At this stage pupils will be working at widely different paces and will need different levels of support, initially with the ICT and later with the mathematics. Consider the use of peer support while setting up the spreadsheet or, if necessary, having a template available.

The teaching notes below give some ideas about how to support pupils with the mathematics, including the use of the 'magic number' and why 5 is placed in the centre square.

When pupils find a solution to the problem, pose questions such as:

Q Are there any other solutions?

Q Can you explain why this total or sum (the magic number) is 15?

Q How many different ways are there to make 15?

Q Why is 5 in the middle square?

Q How do you know you have found all the solutions?

Q What do you notice about the symmetry properties of your solutions?

Plenary

Allow 15 minutes

Mathematics objectives

- Use simple formulae from mathematics and other subjects; substitute positive integers into simple linear expressions and formulae and, in simple cases, derive a formula.
- Present and interpret solutions in the context of the original problem; **explain and justify methods and conclusions**, orally and in writing.
- **Use logical argument to establish the truth of a statement**; give solutions to an appropriate degree of accuracy in the context of the problem. (Year 8)

Ask pupils to share what they have discovered. Pose questions such as:

Q Why are the line totals 15?

Q Which combinations of three numbers make 15? How does that help us?

Q Look at all the solutions and their symmetries. What do you notice?

Q Where are the even numbers? Why do you think that is?

To extend pupils' thinking, you could ask questions like:

Q How you can change the numbers in a magic square to make a 'new' one? For example, could you make one using numbers 11 to 19? … or 0 to 8 ? … or –2 to 6?

Q What are the magic numbers for these squares?

Q How do you work out the magic number? Is there a quick way?

Teaching notes

Teaching strategies

In this lesson the teacher uses a modelling activity to demonstrate how to construct a magic square before pupils create their own. This lesson also provides an opportunity for the first use of a spreadsheet by pupils in Key Stage 3. The activity demonstrates the value of using a spreadsheet to solve problems.

Differentiation

Pupils requiring additional support will benefit from recording some arrangements of the digits from 1 to 9 in 3 by 3 blocks using squared paper and manually finding the total before using the computer. For more able pupils resource 7A2.3, 'More puzzles', provides some challenges for independent working and further development of reasoning skills.

Are there any other solutions?

Note

You only need three numbers (not in a line) to define the magic square. To see what is happening, look at the positions of the 5, 8 and 3.

There is only one solution. All of the following arrangements are obtained by rotating or reflecting the one solution.

8	**3**	4
1	**5**	9
6	7	2

6	1	**8**
7	**5**	**3**
2	9	4

2	7	6
9	**5**	1
4	**3**	**8**

4	9	2
3	**5**	7
8	1	6

4	**3**	**8**
9	**5**	1
2	7	6

8	1	6
3	**5**	7
4	9	2

6	7	2
1	**5**	9
8	**3**	4

2	9	4
7	**5**	**3**
6	1	**8**

One way of clarifying that all of these have the same structure is to join the centre of each square in number order.

Why is the magic number 15?

There are three rows (or columns) which together total

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45

so each must row must total 15.

How many different ways are there to make 15? Why is 5 in the middle? How do you know you have found all the solutions?

Note

Pupils should be encouraged to explain and discuss their reasoning for deciding they have all the solutions.

This table shows all combinations with a total of 15.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1 + 5 + 9 = 15	1				5				9
2 + 4 + 9 = 15		2		4					9
1 + 6 + 8 = 15	1					6		8	
2 + 5 + 8 = 15		2			5			8	
3 + 4 + 8 = 15			3	4				8	
2 + 6 + 7 = 15		2				6	7		
3 + 5 + 7 = 15			3		5		7		
4 + 5 + 6 = 15				4	5	6			
Number of combinations	**2**	**3**	**2**	**3**	**4**	**3**	**2**	**3**	**2**

5 is the only number that appears in four combinations so it is the only number that can go in the centre square because this appears on four lines (one column, one row and two diagonals). 1, 3, 7 and 9 must go on the middle of the sides because they only appear twice.

In each of the four lines that involve the centre square, the numbers are in arithmetic progression, making the magic number equal to 3 times the centre number.

Generating new magic squares

You can create a new magic square from an existing one by adding, subtracting, multiplying or dividing the same number to each entry.

7A2.2 Generalising magic squares

This lesson does not require a computer suite.

Objectives

Mathematics objectives

- Use letter symbols to represent unknown numbers or variables; know the meanings of the words *term*, *expression* and *equation*.
- **Use logical argument to establish the truth of a statement**; give solutions to an appropriate degree of accuracy in the context of the problem. (Year 8)
- Understand that algebraic operations follow the same conventions and order as arithmetic operations.

Lesson outline

Vocabulary

algebra, expression, prove, simplify, symbol, variable

Resources

7A2.3 'More puzzles'

Generalise the magic square to look at other combinations of nine numbers that will make lines with the same total.

Q Make a magic square using a different set of numbers. Do these numbers have to be in sequence?

Q Can you make a magic square whose magic number is 21? 25? 30? 150? 0?

Q What is the connection between the centre number of the magic square and the magic number?

Q Can you prove that a set of nine consecutive numbers can always be arranged to form a magic square? What about other sets of numbers?

Invite pupils to apply what they have learned to:

- develop magic squares using negative numbers;
- look at groups of numbers that could be used to create a magic square;
- create magic squares of 16 numbers.

In the plenary, pupils could report back to the rest of the class what they have discovered.

Teaching notes

Any three sets of three numbers with the following properties will form a 3 by 3 magic square:

- each set of three must be a linear sequence with the same difference, e.g. 1, 7, 13 and 31, 37, 43 and 61, 67, 73 (all have a difference of 6);
- the first terms of the sequences (e.g. 1, 31, 61) must also be a linear sequence (in this case the difference is 30).

Here are two further examples:

- –3, 3, 9 and 27, 33, 39 and 57, 63, 69 (differences: 6 and 30);
- 7, 8, 9 and 13, 14, 15 and 19, 20, 21 (differences: 1 and 6).

This can be proved algebraically as follows.

Let the middle number be n and the line totals be t. Since all the numbers are different, let two of the surrounding numbers be $(n - p)$ and $(n - q)$. We have the following:

$n - q$		$n - p$
	n	

As the total for each diagonal must be $3n$, we can now enter expressions in the bottom two corner cells:

$n - q$		$n - p$
	n	
$n + p$		$n + q$

Because the totals for the first and third columns are also $3n$, we can use informal methods to complete the middle entries:

$n - q$		$n - p$
$n - p + q$	n	$n + p - q$
$n + p$		$n + q$

As each of the top and bottom rows also total $3n$, the remaining entries can be completed:

$n - q$	$n + p + q$	$n - p$
$n - p + q$	n	$n + p - q$
$n + p$	$n - p - q$	$n + q$

The following table contains three sets of three numbers with differences p across and q down:

$n - p - q$	$n - q$	$n - q + p$
$n - p$	n	$n + p$
$n - p + q$	$n + q$	$n + p + q$

7A2.3 More puzzles

This lesson requires pupils to have access to computers.

Objectives

Mathematics objectives

- Use letter symbols to represent unknown numbers or variables; know the meanings of the words *term*, *expression* and *equation*.
- **Use logical argument to establish the truth of a statement**; give solutions to an appropriate degree of accuracy in the context of the problem. (Year 8)
- Understand that algebraic operations follow the same conventions and order as arithmetic operations.

ICT objectives

- Develop ICT-based models and test predictions by changing variables and rules. (Year 8)
- Use software to investigate and amend a simple model by entering rules or formulae and checking their appropriateness and accurate working.

Lesson outline

Resources

Excel file 'It adds up'

One computer for every two or three pupils

Optional: interactive whiteboard

Start the lesson by using the spreadsheet file 'It adds up' with the whole class to clarify the problem. Discuss how pupils might write their own files.

Pupils work in groups on 'It adds up' from **resource 7A2.3** 'More puzzles'.

They could extend this problem by adding more squares to the cross and using the numbers 1–9, 1–13, etc.

The 'Cross cut' problem is a further extension of this idea.

In the plenary, the teacher could draw together what the pupils have discovered and the class could begin to discuss other sorts of puzzles of this type.

Pupils in pairs could choose to work on one (or more) examples from resource 7A2.3 'More puzzles'.

Teaching notes

Activities such as 'It adds up' and 'Cross cut' can help pupils to develop their abilities to reason and prove in mathematics.

It adds up

There are only three basic solutions:

Row total 9

	2	
1	3	5
	4	

Row total 10

	1	
2	5	3
	4	

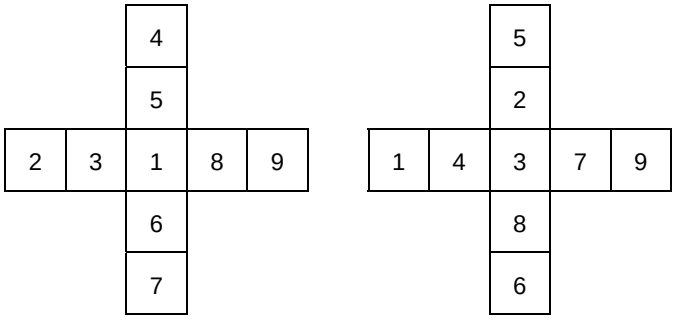
Row total 8

	3	
2	1	5
	4	

If the centre number was even, there would be only one other even number left to place in the 'arms' of the cross. If the second even number was placed in a horizontal arm, the horizontal sum would be odd (even + even + odd = odd) and

the vertical sum even (odd + even + odd = even). A similar argument applies to the second even number in a vertical arm.

Here are two solutions for the cross with nine cells (‘9-cell cross’):

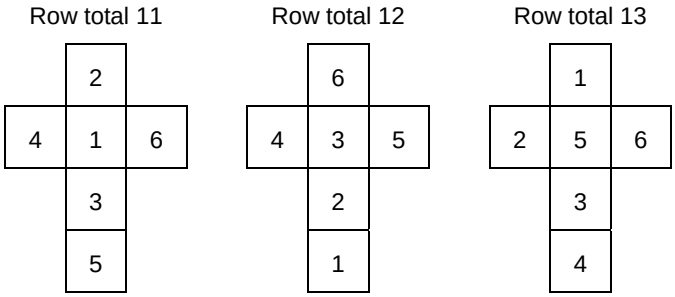


The centre numbers can be any odd number. This then creates a series of line totals from 23 to 27.

For a 13-cell cross the line totals range from 46 to 32.

Cross cut

There are three possible solutions based on the three odd numbers (1, 3, 5) in a 6-cell cross in the centre of the cross.



If the centre number is even (e), there are three odds (O) and two evens (E) left. These can be arranged as follows:

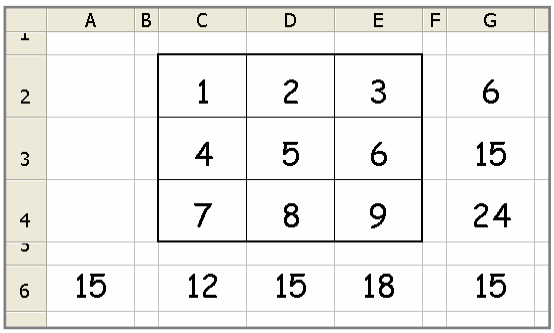
Vertical	Horizontal	Vertical result	Horizontal result
O-e-O-O	E-e-E	O	E
O-e-O-E	O-e-E	E	O
O-e-E-E	O-e-O	O	E

This cross could be extended to make larger crosses.

Building a magic square in Excel

Resource 7A2.1

Start with a blank grid in Microsoft® Excel.
This is what you are going to create.



	A	B	C	D	E	F	G
2			1	2	3		6
3			4	5	6		15
4			7	8	9		24
6	15		12	15	18		15

First, put in all the numbers from 1 to 9 in the grid as shown.

Next enter formulae for the totals of each row, column and diagonal.

How to enter a formula

To enter the formula for the total of the first row:

- Click on cell G2.
- Type: = C2 + D2 + E2
- Press the Enter key.

Excel calculates the total. It should be 6.

Now try changing the *positions* of some of the numbers 1 to 9. Excel will automatically recalculate the totals.

The challenge is to reposition the numbers so that all the totals – three rows, three columns, and two diagonals – are equal.

How many solutions can you find?

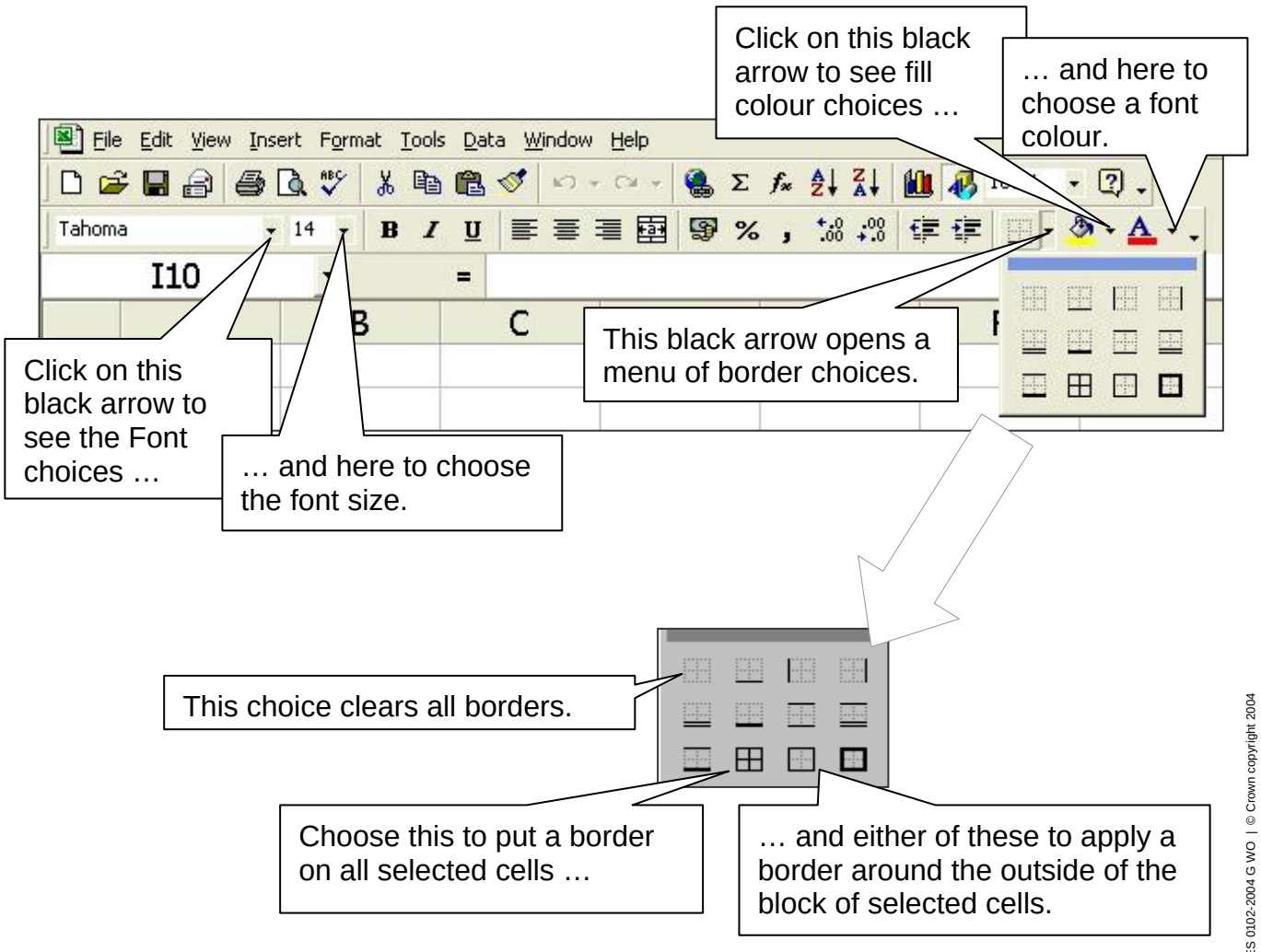
Excel helpsheet: formatting cells

Helpsheet 7A2.2

‘Formatting’ means ‘making choices about appearance’.

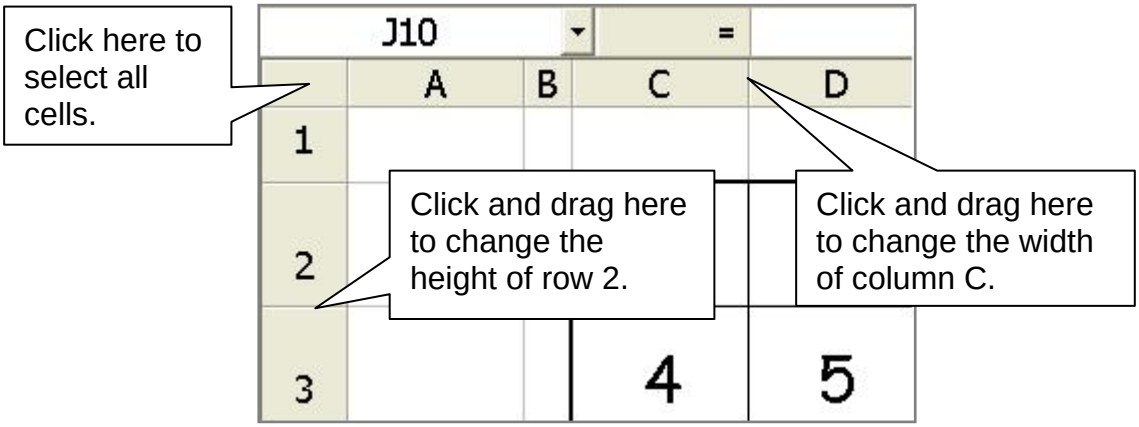
In Microsoft® Excel this includes:

- *alignment* – for example, automatically displaying the number in the middle (centre) of the cell;
- *fill colour* – the cell background colour;
- *borders* – marking the edges of the cells.



Column width and *row height* can also be changed.
A simple way to do this is to click and drag the lower boundary on the row header, or the right hand boundary on the column header.

It can sometimes be very useful before formatting to select all cells.
Click to the left of the column A header to do this.



More puzzles

Resource 7A2.3

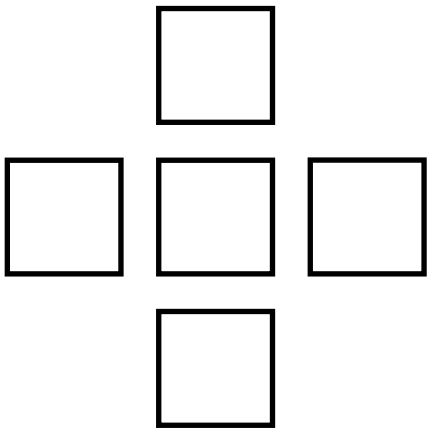
It adds up

Use the numbers 1 to 5, once only.

The sum of the row of 3 must be the same as to the sum of the column of 3.

Prove that the centre number must be odd.

How many different solutions are there?



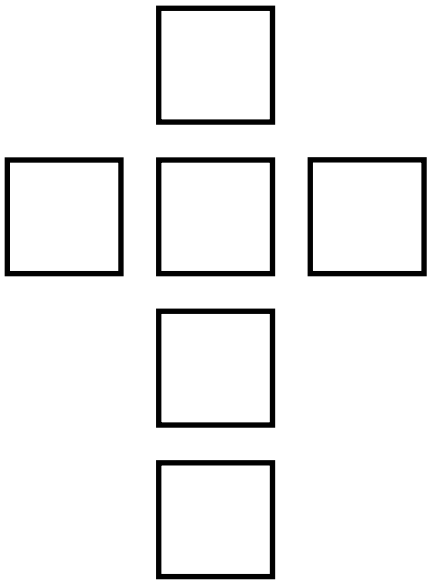
Cross cut

Use the numbers 1 to 6, once only.

The totals of the row of 3 and the column of 4 must be equal.

Prove that the centre number must be odd.

How many different solutions are there?



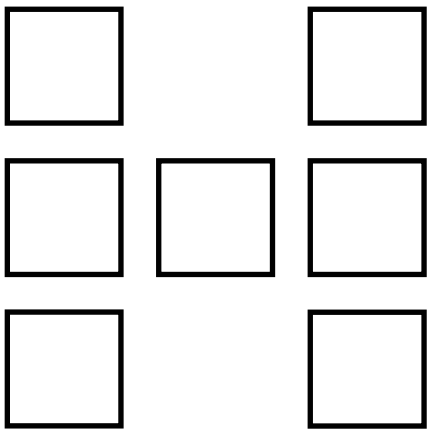
Haitch

Use the numbers 1 to 7, once only so that all three lines have the same total.

How many different possibilities are there?

Prove that 6 and 7 cannot be in the same line.

Prove that the centre of the H must be even.

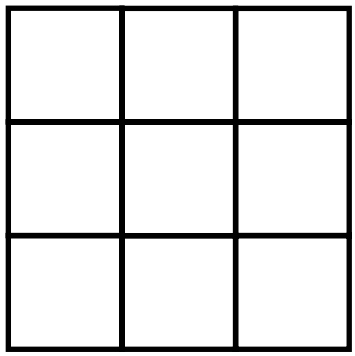


Ten talent

Use any numbers (integers) but each 2 by 2 corner square must add up to 10.

Prove that the four corner numbers in the 3 by 3 square must always add up to an even number.

Is this rule still true if decimals are allowed?



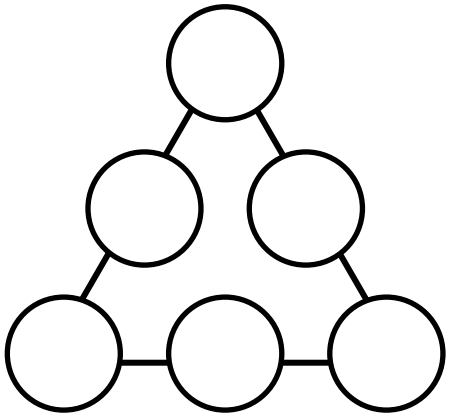
Equalizing

Use the numbers 1 to 6 once only.

Line totals must be equal.

Establish that there are only four solutions.

Prove that the numbers 1 and 6 must be together.

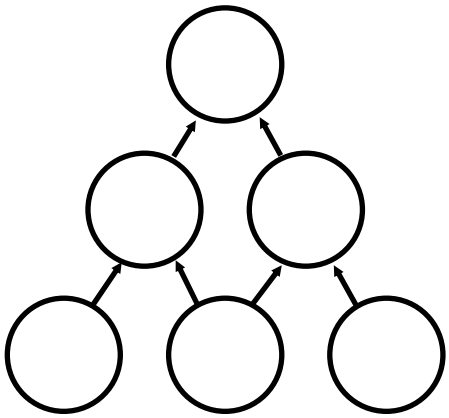


Deduct and deduce

Use the numbers 1 to 6 once only.

Each circle is the positive difference between the two circles below it.

Prove that there are only four different ways of doing this if reflections are not counted.



References and further reading

Framework for teaching mathematics: Years 7, 8 and 9 (DfES 0020/2001)

Framework for teaching ICT capability: Years 7, 8 and 9 (DfES 0321/2002)

ICT and mathematics: a guide to learning and teaching mathematics 11–19 (produced for the Teacher Training Agency by The Mathematical Association, July 2002)

Integrating ICT into mathematics in Key Stage 3 (DfES 0332/2003)

Sample medium-term plans for mathematics (DfES 0504/2001)

Acknowledgements

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